Exam Questions on Normal Distribution

June 2008

A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find $P(X>53)$.

(3)

(b) Find the weight that is exceeded by 99% of the bags.

(5)

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg.

(4)

June 2007

The random variable X has a normal distribution with mean 20 and standard deviation 4.

(a) Find $P(X > 25)$.

(3)

(b) Find the value of d such that $P(20 < X < d) = 0.4641$

(4)

Jan 2007

The measure of intelligence, IQ, of a group of students is assumed to be Normally distributed with mean 100 and standard deviation 15.

(a) Find the probability that a student selected at random has an IQ less than 91.

(4)

The probability that a randomly selected student has an IQ of at least $100 + k$ is 0.2090.

(b) Find, to the nearest integer, the value of k.

(6)

June 2006

From experience a high-jumper knows that he can clear a height of at least 1.78 m once in 5 attempts. He also knows that he can clear a height of at least 1.65 m on 7 out of 10 attempts.

Assuming that the heights the high-jumper can reach follow a Normal distribution,

(a) draw a sketch to illustrate the above information,

(3)

(b) find, to 3 decimal places, the mean and the standard deviation of the heights the high-jumper can reach,

(6)

(c) calculate the probability that he can jump at least 1.74 m.

(3)

Jan 2006

The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg.

Find the probability that a randomly chosen athlete,

(a) is taller than 188 cm,

(3)

(b) weighs less than 97 kg.

(2)

(c) Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg.

(3)

(d) Comment on the assumption that height and weight are independent.

(1)

June 2005

A scientist found that the time taken, M minutes, to carry out an experiment can be modelled by a normal random variable with mean 155 minutes and standard deviation 3.5 minutes.

Find

(*a*) $P(M > 160)$,

(3)

(*b*) $P(150 \leq M \leq 157)$,

(4)

(*c*) the value of m, to 1 decimal place, such that $P(M \leq m) = 0.30$.

(4)

Jan 2005

The random variable X is normally distributed with mean 79 and variance 144.

Find

(*a*) $P(X < 70)$,

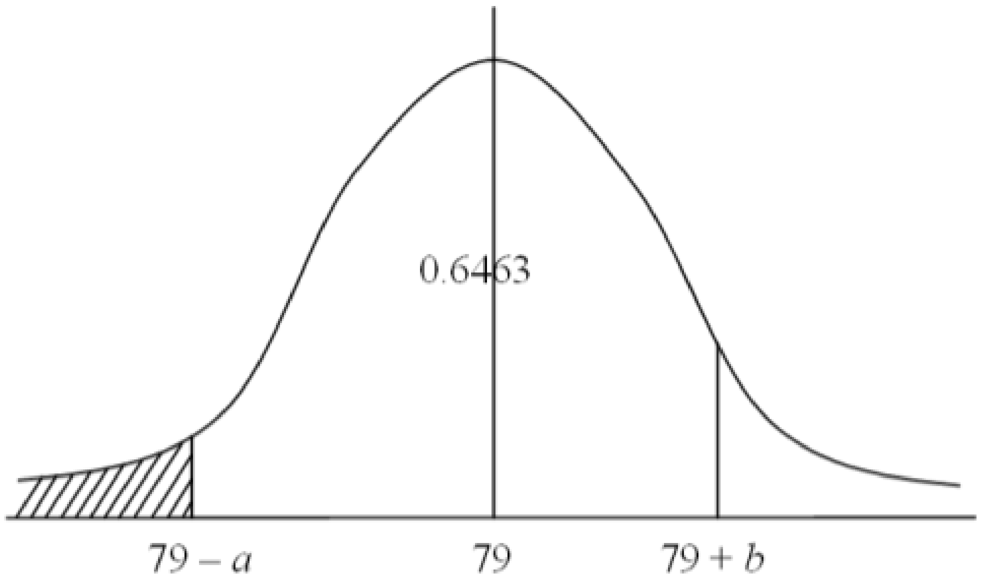
(3)

(*b*) $P(64 < X < 96)$.

(3)

Jan 2005 cont'd

It is known that $P(79 - a \leq X \leq 79 + b) = 0.6463$. This information is shown in the figure below.



Given that $P(X \geq 79 + b) = 2P(X \leq 79 - a)$,

(*c*) show that the area of the shaded region is 0.1179.

(3)

(*d*) Find the value of b.

(4)

June 2004

A health club lets members use, on each visit, its facilities for as long as they wish. The club's records suggest that the length of a visit can be modelled by a normal distribution with mean 90 minutes. Only 20% of members stay for more than 125 minutes.

(*a*) Find the standard deviation of the normal distribution.

(4)

(*b*) Find the probability that a visit lasts less than 25 minutes.

(3)

The club introduce a closing time of 10:00 pm. Tara arrives at the club at 8:00 pm.

(*c*) Explain whether or not this normal distribution is still a suitable model for the length of her visit.

(2)

November 2004

The random variable $X \sim N(\mu, \sigma^2)$.

It is known that

$$P(X \le 66) = 0.0359 \quad \text{and} \quad P(X \ge 81) = 0.1151.$$

(*a*) In the space below, give a clearly labelled sketch to represent these probabilities on a Normal curve.

(1)

(*b*) (i) Show that the value of σ is 5.

(ii) Find the value of μ.

(8)

(*c*) Find $P(69 \le X \le 83)$.

(3)

Jan 2004

The random variable X is normally distributed with mean μ and variance σ^2.

(a) Write down 3 properties of the distribution of X.

(3)

Given that $\mu = 27$ and $\sigma = 10$

(b) find $P(26 < X < 28)$.

(4)

Nov 2003

Cooking sauces are sold in jars containing a stated weight of 500 g of sauce The jars are filled by a machine. The actual weight of sauce in each jar is normally distributed with mean 505 g and standard deviation 10 g.

(*a*) (i) Find the probability of a jar containing less than the stated weight.

(ii) In a box of 30 jars, find the expected number of jars containing less than the stated weight.

(5)

The mean weight of sauce is changed so that 1% of the jars contain less than the stated weight. The standard deviation stays the same.

(*b*) Find the new mean weight of sauce.

(4)

June 2003

The lifetimes of batteries used for a computer game have a mean of 12 hours and a standard deviation of 3 hours. Battery lifetimes may be assumed to be normally distributed.

Find the lifetime, t hours, of a battery such that 1 battery in 5 will have a lifetime longer than t.

(6)

Jan 2003

A drinks machine dispenses coffee into cups. A sign on the machine indicates that each cup contains 50 ml of coffee. The machine actually dispenses a mean amount of 55 ml per cup and 10% of the cups contain less than the amount stated on the sign. Assuming that the amount of coffee dispensed into each cup is normally distributed find

(*a*) the standard deviation of the amount of coffee dispensed per cup in ml,

(4)

(*b*) the percentage of cups that contain more than 61 ml.

(3)

Jan 2003 cont'd

Following complaints, the owners of the machine make adjustments. Only 2.5% of cups now contain less than 50 ml. The standard deviation of the amount dispensed is reduced to 3 ml.

Assuming that the amount of coffee dispensed is still normally distributed,

(c) find the new mean amount of coffee per cup.

(4)

Nov 2002

Strips of metal are cut to length L cm, where $L \sim N(\mu, 0.5^2)$.

(a) Given that 2.5% of the cut lengths exceed 50.98 cm, show that $\mu = 50$.

(5)

(b) Find $P(49.25 < L < 50.75)$.

(4)

Those strips with length either less than 49.25 cm or greater than 50.75 cm cannot be used.

Two strips of metal are selected at random.

(c) Find the probability that both strips cannot be used.

(2)

June 2002

A random variable X has a normal distribution.

(a) Describe two features of the distribution of X.

(2)

A company produces electronic components which have life spans that are normally distributed. Only 1% of the components have a life span less than 3500 hours and 2.5% have a life span greater than 5500 hours.

(b) Determine the mean and standard deviation of the life spans of the components.

(6)

The company gives warranty of 4000 hours on the components.

(c) Find the proportion of components that the company can expect to replace under the warranty.

(4)

Jan 2002

The duration of the pregnancy of a certain breed of cow is normally distributed with mean μ days and standard deviation σ days. Only 2.5% of all pregnancies are shorter than 235 days and 15% are longer than 286 days.

(*a*) Show that $\mu - 235 = 1.96\sigma$.

(2)

(*b*) Obtain a second equation in μ and σ.

(3)

(*c*) Find the value of μ and the value of σ.

(4)

(*d*) Find the values between which the middle 68.3% of pregnancies lie.

(2)

June 2001

The continuous random variable Y is normally distributed with mean 100 and variance 256.

(a) Find $\mathrm{P}(Y < 80)$.

(3)

(b) Find k such that $\mathrm{P}(100 - k \leq Y \leq 100 + k) = 0.516$.

(5)

Jan 2001

The random variable X is normally distributed with mean 177.0 and standard deviation 6.4.

(*a*) Find $P(166 < X < 185)$. **(4 marks)**

It is suggested that X might be a suitable random variable to model the height, in cm, of adult males.

(*b*) Give two reasons why this is a sensible suggestion. **(2 marks)**

(*c*) Explain briefly why mathematical models can help to improve our understanding of real-world problems. **(2 marks)**