S1 Assessment	Normal Distribution	Name:	Date:

Question 1.

The weight of coffee in glass jars labelled 100g is normally distributed with mean 101.5g and standard deviation 7.2g.

The weight of an empty glass is normally distributed with mean 260g and standard deviation 5.3g.

The weight of a glass jar is independent of the weight of the coffee it contains.

Find the probability that a randomly selected jar weighs less than 264g and contains less than 103g of coffee.

[8 marks]

Question 2.

A company produces electronic components which have life spans that are normally distributed. Only 2.5 percent have a life span less than 2975 hours and 1 percent have a life span greater than 5060 hours.

(a) Determine the mean and standard deviation of the life spans of the components.

[6 marks]

There is a warranty of 3770 hours on the components.

(b) Find the proportion of components that the company can expect to replace under the warranty

[4 marks]

Question 3.

The lifetimes of batteries used for a computer game have a mean of 9 hours and a standard deviation of 5 hours.

Battery lifetimes may be assumed to be normally distributed.

Find the lifetime, t hours, of a battery such that 1 battery in 7 will have a lifetime longer than t.

[6 marks]

Question 4.

Strips of metal are cut to length L cm, where $L \sim N(\mu, 0.52)$.

(a) Given that 2 percent of the cut lengths exceed 45.48 cm, show that $\mu = 44.5$, to 1 decimal place.

[5 marks]

(b) Find $P(43.68 < L < 45.32)$.

[4 marks]

Those strips with length less than 43.68 cm or greater than 45.32 cm cannot be used. Two strips of metal are selected at random.

(c) Find the probability that both strips cannot be used.

[2 marks]

Solutions

Question 1.

$$
\begin{aligned}
P(J < 264) &= \varphi\left(\tfrac{264-\mu}{\sigma}\right) = \varphi\left(\tfrac{264-260}{5.3}\right) && |M1A1\\
&= \varphi(0.755)\\
&= 0.7734 && |A1\\
P(C < 103) &= \varphi\left(\tfrac{103-\mu}{\sigma}\right) = \varphi\left(\tfrac{103-101.5}{7.2}\right) && |M1A1\\
&= \varphi(0.208)\\
&= 0.5832 && |A1\\
&\Rightarrow P(J < 264 \text{ and } C < 103) = 0.7734 \times 0.5832 && |M1\\
&\Rightarrow P(J < 264 \text{ and } C < 103) = 0.451 && |A1
\end{aligned}
$$

Question 2

$$
\begin{aligned}
\text{(a)}\quad P(X < 2975) = 0.025 &\Rightarrow \mu - 2975 = 1.96\sigma && |M1A1\\
P(X > 5060) = 0.01 &\Rightarrow \mu - 5060 = -2.327\sigma && |A1\\
&\Rightarrow 2085 = 4.287\sigma\\
&\Rightarrow \sigma = 486.4 && |M1A1\\
&\Rightarrow \mu = 3928.3 && |A1\\
\text{(b)}\quad P(X < 3770) &= \varphi\left(\tfrac{3770-3928.3}{486.4}\right) && |M1A1\\
&= \varphi(-0.325) && |A1\\
&= 0.3707 && |A1
\end{aligned}
$$

Question 3.

$$
\begin{aligned}
\text{Battery lifetime} &\approx N(9, 5^2) && |B1\\
P(B > t) = \tfrac{1}{7} &\Rightarrow P(B < t) = \tfrac{6}{7} && |M1A1\\
&\Rightarrow \varphi\left(\tfrac{t-9}{5}\right) = \tfrac{6}{7} && |M1\\
&\Rightarrow \tfrac{t-9}{5} = 1.068\\
&\Rightarrow t - 9 = 5.34\\
&\Rightarrow t = 14.34 \text{ hours} && |M1A1
\end{aligned}
$$

Question 4.

$$
\begin{aligned}
\text{(a)}\quad P(L > 45.48) = 0.02 &\Rightarrow \tfrac{45.48-\mu}{0.5} = 2.054 && |M2A1\\
&\Rightarrow 45.48 - \mu = 1.027 && |A1\\
&\Rightarrow \mu = 44.453 && |A1\\
\text{(b)}\quad P(43.68 < L < 45.32) &= P(L < 45.32) - P(L < 43.68)\\
&= \varphi\left(\tfrac{45.32-44.5}{0.5}\right) - \varphi\left(\tfrac{43.68-44.5}{0.5}\right) && |M2\\
&= 0.9495 - 0.0505 && |A1\\
&= 0.899 && |A1\\
\text{(c)}\quad \text{P(both not used)} &= (1 - 0.899)^2 = 0.0102 && |M1A1
\end{aligned}
$$